DETERMINATION OF THE ASYMPTOTIC OF THE COMBUSTION WAVE VELOCITY BY SUCCESSIVE APPROXIMATIONS

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Successive approximations can be used not only to study qualitative questions such as the existence of solutions but also to determine the quantitative characteristics of these solutions. Individual papers (see [1]) are known in which this approach was applied to combustion problems but it received no substantial development in combustion theory. This is apparently explained by the fact that a successful realization of the method (there can be several) and the initial approximation has not always been selected successfully. Successive approximations are utilized in this paper to determine the combustion wave velocity in a condensed medium. Convergent approximations are constructed that decrease monotonically to the desired solution as are monotonically increasing approximations. Therefore, upper and lower bounds of the velocity are obtained, where the successful selection of the initial approximation yields agreement between the asymptotics of these bounds for the first approximations, which permits finding the velocity asymptotic in the higher terms.

1. FORMULATION OF THE PROBLEM

The system of differential equations describing n-th order reaction front propagation in a condensed medium has the form

$$\theta'' - u\theta' + (1/\gamma)a^n\Phi(\theta) = 0,\ ua' + (1/\gamma)a^n\Phi(\theta) = 0. \tag{1.1}$$

Here θ is the dimensionless temperature, a is the initial substance concentration, u is the wave velocity; the prime denotes differentiation with respect to the space variable x

$$\Phi(\theta) = \begin{cases} 0, & (-1 \leqslant \theta < -1 + h), \\ \exp\dfrac{\theta}{\gamma + \beta\theta}, & (-1 + h \leqslant \theta \leqslant 0); \end{cases} \tag{1.2}$$

β and γ are traditional small parameters for combustion problems, $\beta = RT_*/E$; $\gamma = RT_*^2/qE$; T_* is the combustion temperature; $T_* = T_i + q$, T_i is the initial mixture temperature, q is adiabatic heating of the reaction, E is the activation energy of the reaction, R is the gas constant, and h is the magnitude of the source pieces. The boundary conditions as $x \to \pm\infty$ are: $\theta(-\infty) = -1$, $a(-\infty) = 1$, $\theta(+\infty) = a(+\infty) = 0$.

The asymptotic of the combustion wave velocity was considered for the model mentioned in [2-5]. At the present time it can be considered that the question of the determination of the two highest terms of a nonuniform asymptotic is solved for $n < 2$. We understand the nonuniformity of the asymptotic in the following sense: The limit of the asymptotic in the infinitesimal γ equals zero as $n \to 2$ and does not agree with the asymptotic in γ obtained for $n = 2$. Such a situation is characteristic for the case of successive application of the method of merging of asymptotic expansions in powers of γ. Successive approximations permits construction of a uniform asymptotic for $n \leq 2$.

2. REALIZATION OF SUCCESSIVE APPROXIMATIONS FOR $0 \leq n \leq 1$

The system of equations (1.1) has a first integral and can be reduced in the ordinary way to one equation

$$\frac{da}{d\theta} = -\frac{1}{\gamma u^2}\frac{a^n \exp\dfrac{\theta}{\gamma+\beta\theta}}{a(\theta)+\theta} \tag{2.1}$$

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with the boundary conditions

$$a(-1+h) = 1,\ a(0) = 0 \tag{2.2}$$

(θ is a new independent variable, $-1 + h \le \theta \le 0$, $a(\theta)$ is an unknown function). It is easy to obtain from (2.1) and (2.2)

$$a^{2-n}(\theta) = 1 - \frac{2-n}{\gamma u^2} \int\limits_{-1+h}^{\theta} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau/a}\, d\tau; \tag{2.3}$$

$$u^2 = \frac{2-n}{\gamma} \int\limits_{-1+h}^{0} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau/a}\, d\tau. \tag{2.4}$$

Let us set

$$a_{i+1}^{2-n}(\theta) = 1 - \frac{2-n}{\gamma u^2} \int\limits_{-1+h}^{\theta} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau/a_i(\tau)}\, d\tau, \quad a_0(\theta) \equiv 1. \tag{2.5}$$

Since $a_0(\theta) \geqslant a(\theta)$ for $-1 + h \le \theta \le 0$, then $a(\theta) \leqslant a_1(\theta) \leqslant a_0(\theta)$ for $-1 + h \le \theta \le 0$ by virtue of (2.3) and (2.5). Hence, by induction

$$a(\theta) \leqslant a_{i+1}(\theta) \leqslant a_i(\theta)\ (-1+h \leqslant \theta \leqslant 0). \tag{2.6}$$

Let $\tilde{a}(\theta)$ denote the limit of the sequence of functions $\{a_i(\theta)\}$. Passing to the limit in (2.5) [it is possible to pass to the limit under the integral sign by the Lebesgue theorem because of the inequality (2.6)], we find that $\tilde{a}$ satisfies (2.3) and, therefore, (2.1). It is also clear that if u is the wave velocity then $\tilde{a}(-1+h) = 1$, $\tilde{a}(0) = 0$.

Therefore, a decreasing sequence of functions has been obtained that converges to the solution and a corresponding sequence of inequalities for the velocity is

$$u^2 > F_i(u^2) \equiv \frac{2-n}{\gamma} \int\limits_{-1+h}^{0} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau/a_i(\tau)}\, d\tau \quad (i = 0, 1, 2, \ldots), \tag{2.7}$$

whose right sides are functions of u^2. It is seen that F_i decrease monotonically in u^2 and $F_{i+1} > F_i$. If u_i^2 denotes the solution of the equation

$$u^2 = F_i(u^2), \tag{2.8}$$

then $a_{i+1}(0) = 0$ for the functions (2.5), where u_i is substituted in place of u. Consequently, $F_{i+1}(u^2)$ is defined for $u^2 > u_i^2$. Therefore, the solution of (2.8) exists, is unique, and the sequence of numbers $\{u_i\}$ converges to the value of the velocity by growing.

Let us note that the successive approximations (2.5) are determined and converge to the solution for any values of $n \ge 0$; however, the expression for the velocity (2.4) and the inequality (2.7) hold only for $n < 2$.

Let us consider the function $\alpha_i(\theta)$ in addition to the functions $a_i(\theta)$:

$$\alpha_i^{2-n}(\theta) = \frac{2-n}{\gamma u^2} \int\limits_{\theta}^{0} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau/a_i(\tau)}\, d\tau \quad (i = 0, 1, 2, \ldots). \tag{2.9}$$

From the representation of the solution

$$a^{2-n}(\theta) = \frac{2-n}{\gamma u^2} \int\limits_{\theta}^{0} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau/a(\tau)}\, d\tau \tag{2.10}$$

and the inequalities (2.6)

$$\alpha_i(\theta) \leqslant \alpha_{i+1}(\theta) \leqslant a(\theta),\ i = 0, 1, 2, \ldots\ (-1+h \leqslant \theta \leqslant 0). \tag{2.11}$$

The existence of the integral in (2.9) follows from the fact that the integral in (2.10) is defined, and from the inequalities (2.6).

Using the representation (2.4) for the velocity and the inequalities (2.11), we obtain

$$u^2 < \frac{2-n}{\gamma} \int\limits_{-1+h}^{0} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau/\alpha_i(\tau)} d\tau \quad (i=0,1,2,\ldots). \tag{2.12}$$

The existence of the integral in (2.12) for all i evidently results from the existence of the integral for i = 0. For this it is sufficient to require that the following conditions be satisfied

$$\alpha_0(\theta) > -\theta \quad (0 > \theta \geqslant -1+h) \tag{2.13}$$

and

$$\lim_{\theta\to 0} (-\theta/\alpha_0(\theta)) < 1. \tag{2.14}$$

For $n > 1$ (2.13) is spoiled near zero. In the case $n \leq 1$ that is examined in this section, it follows from the inequality $\alpha_0{}^{2-n}(\theta) > -\theta$, i.e.,

$$J_0(\theta) \equiv \frac{2-n}{\gamma u^2} \int\limits_{\theta}^{0} \frac{\exp \frac{\tau}{\gamma+\beta\tau}}{1+\tau} d\tau > -\theta \quad (-1+h \leqslant \theta < 0).$$

It is easy to confirm that $J_0''(\theta) < 0$ for $\gamma < h$. Since $J_0(0) = 0$ and $J_0(-1+h) \to 1$ as $\gamma \to 0$ ($u^2 \to 2-n$ as $\gamma \to 0$ [6]), then (2.13) is satisfied for sufficiently small γ. The value of the limit in (2.4) equals zero for $n < 1$ and γu^2 for $n = 1$.

Let $\Phi_i(u^2)$ denote the right side in (2.12). It is seen that the functions a_{i+1} and α_i agree for $u = u_i$, consequently, $\Phi_i(u_i{}^2) = F_{i+1}(u_i{}^2)$. Furthermore, the functions $\Phi_i(u^2)$ are growing and $\Phi_{i+1} < \Phi_i$ in the domain of definition. If $\tilde{u}_i{}^2$ denotes the solution of the equation

$$u^2 = \Phi_i(u^2) \tag{2.15}$$

(more exactly, the least of the solutions), then we find a decreasing sequence of numbers $\tilde{u}_i{}^2$ that converges to the square of the velocity. The solvability of (2.15) for $i > 0$ follows from the solvability for $i = 0$ which holds for sufficiently small γ as will be seen later.

3. ASYMPTOTIC OF THE VELOCITY FOR $0 \leq n \leq 1$

It is shown above that the inequality

$$F_1(u^2) < u^2 < \Phi_0(u^2)$$

is satisfied. In this section two of the highest terms of the asymptotic expansion of the functions F_1 and Φ_0 as $\gamma \to 0$ are presented. Since these expansions agree, the asymptotic of the velocity can be obtained. Let us note that a larger number of terms of the expansion will agree for the functions F_i and Φ_{i-1} ($i > 1$), which yields the next terms of the velocity asymptotic. The estimate (2.7) for $i = 0$ agrees with the estimate obtained by the minimax method [6] while the function $a_1(\theta)$ agrees with the main trial function used there.

Let us represent the function F_1 in the form $F_1 = F_{11} + F_{12}$, where

$$F_{11} = (2-n) \int\limits_{(-1+h)/\gamma}^{0} \exp \frac{\theta}{1+\beta\theta} d\theta;$$

$$F_{12} = (2-n)\gamma \int\limits_{(-1+h)/\gamma}^{0} \frac{-\theta \exp \frac{\theta}{1+\beta\theta} d\theta}{\gamma\theta + \left[1 - \frac{2-n}{u^2} \int\limits_{(-1+h)/\gamma}^{\theta} \frac{\exp \frac{\tau}{1+\beta\tau}}{1+\gamma\tau} d\tau\right]^{1/(2-n)}}.$$

Let us make the change of variables $x = \beta^{-1}(1+\beta\theta)^{-1}$ under the integral sign in the expression for F_{11} and let us use the asymptotic representation of the incomplete Gamma function

$$\int\limits_{(-1+h)/\gamma}^{0} \exp\frac{\theta}{1+\beta\theta}\,d\theta = \frac{1}{\beta^2}\exp\frac{1}{\beta}\left[-\frac{e^{-x}}{x}\Big|_{x=1/\beta}^{x=\frac{1/\beta}{1+\beta(-1+h)/\gamma}},\right.$$

$$\left.-\Gamma\left(0,\frac{1}{\beta}\right)+\Gamma\left(0,\frac{1/\beta}{1+\beta\frac{-1+h}{\gamma}}\right)\right] = 1-2\beta+o(\gamma).$$

Let I_{12} denote the integral in the expression F_{12}. If we pass to the limit formally therein as $\gamma \to 0$, then

$$\lim_{\gamma\to 0} I_{12} = \int\limits_{-\infty}^{0} \frac{-\theta\exp\theta}{(1-\exp\theta)^{1/(2-n)}}\,d\theta, \tag{3.1}$$

and the asymptotic representation for the function F_1 has the form

$$F_1 = (2-n)\left[1-2\beta-\gamma\int\limits_{-\infty}^{0}\frac{\theta\exp\theta\,d\theta}{(1-\exp\theta)^{1/(2-n)}}+o(\gamma)\right].$$

To give a foundation to the passage to the limit (3.1), the integral I_{12} must be separated into two integrals with the limits from $(-1 + h)/\gamma$ to $-N$ and from $-N$ to 0. For sufficiently large N and small γ the former of them is a small quantity while the passage to the limit $\gamma \to 0$ can be made in the second integral with constant limits by the Lebesgue theorem.

Analogously, we find the asymptotic representation of the function $\Phi_0(u^2)$. Let us note that if the convergence $u^2 \to 2 - n$ as $\gamma \to 0$, is not used, then for fixed u^2

$$\Phi_0(u^2) = (2-n)\left[1-2\beta+\gamma\left(\frac{u^2}{2-n}\right)^{1/(2-n)}\int\limits_{-\infty}^{0}\frac{-\theta\exp\theta\,d\theta}{(1-\exp\theta)^{1/(2-n)}}+o(\gamma)\right].$$

In particular, there hence follows the solvability of (2.15) for small γ, i = 0. Therefore, for $0 \le n \le 1$ the two highest terms of the upper and lower bound asymptotics agree, therefore, the velocity asymptotic has the form

$$u^2 = (2-n)\left(1-2\beta+\gamma\int\limits_{0}^{\infty}\frac{x\exp(-x)\,dx}{[1-\exp(-x)]^{1/(2-n)}}+o(\gamma)\right). \tag{3.2}$$

The coefficient of γ can also be represented as

$$\int\limits_{0}^{\infty}\frac{x\exp(-x)\,dx}{[1-\exp(-x)]^{1/(2-n)}} = \frac{2-n}{n-1}\left[\Psi(1)-\Psi\left(\frac{3-2n}{2-n}\right)\right],$$

where $\Psi(z) = \Gamma'(z)/\Gamma(z)$; $\Gamma(z)$ is the Gamma function and $\Psi(z)$ is the diagram function.

For $0 \le n \le 1$ the velocity asymptotic (3.2) agrees with the asymptotic obtained in [2, 4] by merging the asymptotic expansions.

4. SUCCESSIVE APPROXIMATIONS FOR $1 \le n \le 2$

The initial approximation examined in Sec. 3 does not permit obtaining the asymptotic of the upper bound for $n > 1$. Consequently, we give another initial approximation. Let us consider the equation

$$\frac{db}{d\theta} = -\frac{1}{\gamma u^2}\frac{b^n}{b+\theta}. \tag{4.1}$$

Equating (2.1) and (4.1), we see that if $b(\theta) = a(\theta)$ then $db/d\theta < da/d\theta$, i.e., the trajectories of (4.1) intersect the trajectories of (2.1) from the top down. Consequently, the solution of (4.1) with the boundary conditions $b(0) = 0$ becomes equal to one for a certain $\theta = \theta_0$. We take the function $b_0(\theta)$ that agrees with the mentioned solution when it is less than one and equal to one for $-1 + h \le \theta \le \theta_0$ as the initial approximation. Therefore

$$a(\theta) \leqslant b_0(\theta) \leqslant 1 \quad (-1+h \leqslant \theta \leqslant 0) \tag{4.2}$$

[$a(\theta)$ is a solution of the problem (2.1) and (2.2)]. As before, we give the subsequent approximations in the form

$$b_{i+1}^{2-n}(\theta) = 1 - \frac{2-n}{\gamma u^2} \int_{-1+h}^{\theta} \frac{\exp \frac{\tau}{\gamma + \beta\tau}}{1 + \tau/b_i(\tau)} d\tau, \quad i = 0, 1, 2, \ldots$$

It follows from (4.2) that $a(\theta) \leqslant b_i(\theta) \leqslant a_i(\theta)$, $i = 0, 1, 2, \ldots$ $(-1 + h \leqslant \theta \leqslant 0)$, and hence the convergence of the successive approximations $b_i(\theta)$ to the solution $a(\theta)$ and the inequality

$$u^2 > \frac{2-n}{\gamma} \int_{-1+h}^{0} \frac{\exp \frac{\tau}{\gamma + \beta\tau}}{1 + \tau/b_i(\tau)} d\tau, \quad i = 0, 1, 2, \ldots$$

In order to obtain the upper bound of the velocity, one more sequence of functions bounding the solution from below

$$\beta_i^{2-n}(\theta) = \frac{2-n}{\gamma u^2} \int_{\theta}^{0} \frac{\exp \frac{\tau}{\gamma + \beta\tau}}{1 + \tau/b_i(\tau)} d\tau, \quad i = 0, 1, 2, \ldots$$

must be given. If

$$\beta_i(\theta) > -\theta \ (-1 + h \leqslant \theta < 0), \tag{4.3}$$

then

$$u^2 < \frac{2-n}{\gamma} \int_{-1+h}^{0} \frac{\exp \frac{\tau}{\gamma + \beta\tau}}{1 + \tau/\beta_i(\tau)} d\tau, \quad i = 0, 1, 2, \ldots . \tag{4.4}$$

Let us note that finiteness of the integral in (4.4) will does not result from (4.3). Since the inequality (4.4) will be used later for i = 0, it must be confirmed that it is satisfied in this case.

For $\theta \le \theta_0$, $b_0(\theta) \equiv 1$ and the inequality (4.3) is proved exactly as is (2.13). The function β_0 satisfies the equation

$$\frac{d\beta_0}{d\theta} = -\frac{1}{\gamma u^2} \frac{\beta_0^{n-1} b_0 \exp \frac{\theta}{\gamma + \beta\theta}}{b_0 + \theta} \tag{4.5}$$

with the boundary condition $\beta_0(0) = 0$. Consequently, the validity of (4.3) for $\theta_0 \le \theta < 0$ follows from the inequality

$$-\frac{1}{\gamma u^2} \frac{(-\theta)^{n-1} b_0 \exp \frac{\theta}{\gamma + \beta\theta}}{b_0 + \theta} < -1, \tag{4.6}$$

which means that the trajectories of Eqs. (4.5) intersect the line $\beta_0 = -\theta$ from the top down. Introducing the notation

$$\varepsilon = (\gamma\sigma m)^m/\gamma, \ \psi_m(z) = z^{1-m} E_m(z) \exp z, \quad m = 1/(n-1), \ z = \gamma\sigma m [b_0(\theta)]^{-1/m} \tag{4.7}$$

(E_m is the exponential integral function), we find $\theta/b_0(\theta) = -zE_m(z)\exp z$ by solving (4.1) and we represent (4.6) as

$$m [\psi_m(z)]^{1/m} \exp \frac{-\varepsilon\psi_m(z)}{1 - \beta\varepsilon\psi_m(z)} \geqslant 1 - zE_m(z) \exp z \quad (z \geqslant \gamma\sigma m). \tag{4.8}$$

The validity of (4.8) is confirmed by simple calculations in which the inequalities $1/(z + m) \le E_m(z) \exp z$ $(z \ge 0, m > 1)$ and $\Psi_m(z) \le -\theta_0/\gamma\varepsilon$ $(z > \gamma\sigma m)$, resulting from the properties of these special functions must be taken into account.

5. VELOCITY ASYMPTOTIC FOR 1 < n ≤ 2

As in Sec. 3, we obtain the asymptotic representations of the upper and lower bounds.

Let J denote the right side in the estimate (4.4) for i = 0 and after simple identity transformations using the notation (4.7), we write

$$J=(2-n)\left[\int\limits_{(-1+h)/\gamma}^{0}\exp\frac{\theta}{1+\beta\theta}\,d\theta+J_1+J_2\right],$$

where

$$J_1=-\gamma\int\limits_{(-1+h)/\gamma}^{\theta_0/\gamma}\left[\theta\exp\frac{\theta}{1+\beta\theta}\Big/\left\{\gamma\theta+\left[\frac{2-n}{u^2}\left(\int\limits_{\theta}^{0}\exp\frac{\tau}{1+\beta\tau}\,d\tau-\right.\right.\right.\right.$$

$$\left.\left.\left.\left.-\gamma\int\limits_{\theta}^{\theta_0/\gamma}\frac{\tau\exp\frac{\tau}{1+\beta\tau}}{1+\gamma\tau}\,d\tau+\varepsilon\int\limits_{\gamma u^2 m}^{\infty}\psi_m(z)\exp\frac{-\varepsilon\psi_m(z)}{1-\beta\varepsilon\psi_m}\,dz\right)\right]^{1/(2-n)}\right\}\right]d\theta; \tag{5.1}$$

$$J_2=-\varepsilon\int\limits_{\theta_0/\gamma\varepsilon}^{0}\left[\tau\exp\frac{\varepsilon\tau}{1+\beta\varepsilon\tau}\Big/\left\{\tau+(m-1)^{m/(m-1)}\left[\frac{1}{\varepsilon}\int\limits_{\varepsilon\tau}^{0}\exp\frac{\tau_1}{1+\beta\tau_1}\,d\tau_1+\right.\right.\right.$$

$$\left.\left.\left.+\int\limits_{z_0(\tau)}^{\infty}\psi_m(z)\exp\frac{-\varepsilon\psi_m(z)}{1-\beta\varepsilon\psi_m}\,dz\right]^{m/(m-1)}\right\}\right]d\tau. \tag{5.2}$$

Therefore, the question of determining the asymptotic of the upper bound reduces to determining the asymptotics of the integrals J_1 and J_2. It is simple to show that

$$J_1=-\gamma\int\limits_{-\infty}^{-1}\frac{\theta\exp\theta}{(1-\exp\theta)^{1/(2-n)}}\,d\theta+o(\gamma); \tag{5.3}$$

$$J_2=-\gamma\int\limits_{-1}^{0}\frac{\theta\exp\theta}{(1-\exp\theta)^{1/(2-n)}}\,d\theta+o(\gamma)\quad(1<n<3/2); \tag{5.4}$$

$$J_2=-\varepsilon\int\limits_{-\infty}^{0}\frac{\tau\,d\tau}{\tau+(m-1)^{m/(m-1)}\left[-\tau+\int\limits_{z_0(\tau)}^{\infty}\psi_m(z)\,dz\right]^{m/(m-1)}}+o(\varepsilon)= \tag{5.5}$$

$$=\varepsilon\int\limits_{0}^{\infty}\psi_m(z)\,dz+o(\varepsilon)=\varepsilon\frac{\Gamma(2-m)}{m-1}+o(\varepsilon)\quad\left(\frac{3}{2}<n<2\right).$$

It is taken into account in (5.3) and (5.4) that $u^2\to 2-n$ as $\gamma\to 0$, which does not affect the form of the highest terms. It is generally impossible to make such a replacement in (5.5) since precisely it results in nonuniformity of the asymptotic. If, however, we have a nonuniform asymptotic in mind, then by virtue of (5.3)-(5.5) it is written for the upper bound as

$$\bar{u}^2=(2-n)\left[1-2\beta+\gamma\int\limits_{0}^{\infty}\frac{x\exp(-x)\,dx}{(1-\exp(-x))^{1/(2-n)}}+o(\gamma)\right]\quad\left(1<n<\frac{3}{2}\right); \tag{5.6}$$

$$\bar{u}^2=(2-n)\left[1+\gamma^{\frac{2-n}{n-1}}\left(\frac{2-n}{n-1}\right)^{\frac{2-n}{n-1}}\Gamma\left(\frac{2n-3}{n-1}\right)+o\left(\gamma^{\frac{2-n}{n-1}}\right)\right]\quad\left(\frac{3}{2}<n<2\right). \tag{5.7}$$

For n = 3/2 the expressions (5.6) and (5.7) cannot be used since the magnitude of the discarded terms depends on n and grows as $n\to 3/2$. The asymptotic of the integrals (5.1) and (5.2) for n = 3/2 yields

$$\bar{u}^2=(1/2)(1-\gamma\ln\gamma+o(\gamma))\ (n=3/2). \tag{5.8}$$

The asymptotic of the lower bound is obtained analogously and agrees with (5.6)-(5.8). Therefore, the nonuniform velocity asymptotic for 1 < n < 2 has the form (5.6)-(5.8) and agrees with that obtained in [2, 5].

By limiting ourselves to the highest term in the expansions of the integrals J_1 and J_2 in powers of γ, it is impossible to obtain a general representation for the velocity asymptotic for $1 < n < 2$, as is seen from the expressions presented above. The mentioned general representation for the upper bound of the velocity will be

$$\bar{u}^2 = (2-n)\left[1 - 2\beta - \frac{\varepsilon_0 \psi_{m-2}(\gamma(m-1))}{(m-2)(m-1)} + \frac{\varepsilon_0}{(m-2)(m-1)^{1/(m-1)}} + \right. \left. + \frac{1}{(m-1)^{m/(m-1)}} \sum_{r=1}^{\infty} \frac{\varepsilon_0^{1/(m-1)} - \varepsilon_0^r}{r(r - 1/(m-1))} + o\left(\left(\frac{\varepsilon_0 - \gamma}{2-m}\right)^2\right)\right], \tag{5.9}$$

where $\varepsilon_0 = \gamma^{m-1}(m-1)^m$. Let us note that the asymptotic (5.9) is determined to the accuracy of quadratic terms, which results in the appearance of additional terms of the expansion as compared with (5.6)-(5.8). It cannot be simplified substantially for all n, $1 < n \le 2$.

An analogous representation can be written also for the lower bound asymptotic; however only the two highest terms of the expansion agree for them.

In conclusion, let us examine the question of a uniform velocity asymptotic as $n \to 2$. As already mentioned, successive approximation affords the possibility of obtaining velocity upper and lower bounds in terms of functions that are, in turn, dependent on the velocity. The agreement between the two highest terms of the asymptotics of these functions results in an asymptotic equality for the velocity ($3/2 < n < 2$)

$$u^2 = (2-n)\left[1 + \left(\frac{\gamma u^2}{n-1}\right)^{\frac{2-n}{n-1}} \Gamma\left(\frac{2n-3}{n-1}\right) \frac{u^2}{2-n} + o\left(\gamma^{\frac{2-n}{n-1}} u^{\frac{2}{n-1}}\right)\right]. \tag{5.10}$$

Analysis of the integrals J_1 and J_2 shows that the discarded terms remain bounded as $n \to 2$, which permits passage to the limit. Consequently, $u^2 \ln(1/\gamma u^2) \sim 1$ for $n = 2$ for which $u^2 \sim 1/\ln(1/\gamma)$, which agrees with the highest term of the asymptotic in [2]. Neglecting the smallest terms in (5.10), we obtain

$$u^2 = \frac{2-n}{1 - \left(\frac{\gamma u^2}{n-1}\right)^{\frac{2-n}{n-1}} \Gamma\left(\frac{2n-3}{n-1}\right)}.$$

Solving this transcendental equation for u^2 by successive approximations, we find

$$u^2 \sim \frac{2-n}{1 - \gamma^{\frac{2-n}{n-1}} \left(\frac{2-n}{n-1}\right)^{\frac{2-n}{n-1}} \Gamma\left(\frac{2n-3}{n-1}\right) \frac{1}{(1-\gamma^{2-n})^{2-n}}}, \tag{5.11}$$

which yields a uniform asymptotic as $n \to 2$ and the two highest terms of the nonuniform asymptotic for $3/2 < n < 2$.

LITERATURE CITED

1. F. A. Williams, Combustion Theory, Addison-Wesley, MA (1965).
2. V. S. Berman and Yu. S. Ryazantsev, "Asymptotic analysis of the propagation of the front of an exothermal single-stage n-th order reaction in a condensed phase, Fiz. Goren. Vzryva, No. 1 (1975).
3. V. N. Galkina, V. I. Lyubchenko, and G. N. Marchenko, "On the velocity of combustion wave propagation in a condensed phase," Dokl. Akad. Nauk SSSR, 286, No. 2 (1986).
4. S. I. Khudyaev, "On the asymptotic theory of a stationary combustion wave," Khim. Fiz., No. 5 (1987).
5. S. I. Khudyaev and A. M. Il'in, "On the asymptotic of a stationary combusion wave in a condensed medium," Khim. Fiz., No. 4 (1989).
6. V. A. Vol'pert, V. A. Vol'pert, and D. S. Davtyan, Estimates of the Combustion Wave Velocity in a Condensed Phase [in Russian], Preprint, Akad. Nauk SSSR, Otdelenie Inst. Khim. Fiz., Chernogolovka (1988).